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INFORMATION AGGREGATION, GROWTH, AND FRANCHISE EXTENSION WITH APPLICATIONS TO FEMALE ENFRANCHISEMENT AND INEQUALITY

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ABSTRACT

We develop a model of voluntary gradual franchise extension and growth based on the idea that voting is an information aggregation mechanism. A larger number of voters means that more correct decisions are made, hence increasing output, but also implies that any incremental output must be shared among more individuals. These conflicting incentives lead to a dynamic model of franchise extensions that is consistent with several real world episodes, including female enfranchisement. The model also predicts that in certain circumstances growth and enfranchisement will be accompanied by Kuznets curve type behaviour in inequality. Contrary to the preceding literature these conclusions do not rest on incentives for strategic delegation.

Keywords: democracy, franchise extension, growth

JEL classification numbers: H0, P4, P16

I. INTRODUCTION

There is now an extensive literature addressing the seemingly paradoxical question of why elites sometimes choose to dilute their own power by extending the franchise. Examples include Acemoglu and Robinson (2006), Lizzeri and Persico (2004), Justman and Gradstein (1999), Llavador and Oxoby (2005), Conley and Temimi (2001), and Ellis and Fender (2009, 2011, 2014). In an important paper, Jack and Lagunoff (2006) argue that many of the models that have tried to explain this phenomenon share certain key features. Most importantly they involve strategic delegation, in the sense that the current pivotal individual in an elite may have an incentive to extend the franchise so as to make another individual decisive, and by doing so can commit to future policies that would not otherwise have been credible. However, this incentive to expand the franchise is tempered by the knowledge that the new pivotal individual faces the same incentives and will subsequently behave similarly. Jack and Lagunoff develop a quite

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general recursive game that is compatible with several of the proposed explanations of voluntary franchise extensions. Furthermore, the dynamics of franchise extension generated by the model are consistent with the 'stylized facts' that: (1) most extensions are partial; (2) extensions are typically gradual processes; and (3) extensions are uneven. In support of these facts, Jack and Lagunoff muster an impressive body of evidence, including examples from the Greek city states, the Roman Republic in the period 494BC to 287BC, and the more commonly known examples of England, Prussia, and the Netherlands in the nineteenth century. A broad distinction can be made in this strategic delegation literature between two types of explanations for franchise extensions. The first is where the elite is subject to a credible revolutionary threat.[1] Franchise extension occurs as a way of committing the elite to redistributive policies necessary to avert the revolution. The second is where there are divisions within the elite, and the section of the elite that is currently dominant finds it in its interest to extend the franchise (perhaps because a majority of the newly enfranchised would support policies it favours).[2]

In this paper we present a model of gradual voluntary franchise extension that is also consistent with the stylized facts but is quite different from the strategic delegation story; indeed in our analysis we initially assume all voters are identical and each, if pivotal, would make identical decisions at each point in time, so there is no role whatsoever for strategic delegation.[3] The key difference is that we model voting as an information aggregation mechanism in the spirit of Condorcet's 'Jury Theorem' (Condorcet, 1785) rather than as a mechanism for arbitrating conflicts of interest. The two approaches, we believe, are quite complementary, and both do a respectable job in explaining many actual episodes of franchise extension. However, we would argue that our approach, suitably extended to incorporate agent heterogeneity of some sort, can provide a persuasive explanation for many countries' decisions to extend the franchise to women, something which is extremely difficult to do using any version of the strategic delegation approach mentioned above. Of course there are many franchise extensions where strategic delegation explanations are plausible, so we see our approach as providing an *additional* (rather than an alternative) reason why an elite might extend the franchise.

The Condorcet Jury Theorem (for a review article and references, see Piketty, 1999) states, basically, that more voters are more likely to make correct decisions. It is perhaps the most fundamental result about why democracies may, on average, make better decisions than less inclusive governance structures: according to Piketty (1999, p. 794): 'This informational efficiency result about majority-rule voting should be given the same status in political theory as the Arrow–Debreu efficiency result in economic theory: it provides us with the most basic (and most fundamental) rationale for the most basic political institution'. The underlying idea is that if each voter has information that has value even though it might be extremely noisy, and this information is independent across voters, then the mean of these voters' information is likely to be much more accurate than any individual's, and this accuracy increases as the number of voters increases. Asymptotically, as the number of voters reaches infinity, the probability of reaching the correct decision is unity. The theorem is essentially a consequence of the law of large numbers.

[1] Acemoglu and Robinson (2006) and Ellis and Fender (2009, 2011, 2014) are all examples of this type of explanation.

[2] Lizzeri and Persico (2004) and Llavador and Oxoby (2005) are prominent examples of explanations of this type.

[3] Strictly, we should say that each individual does have some idiosyncratic information, but is otherwise the same as everyone else, so there is no role for strategic delegation. The reason we first of all assume identical agents is to highlight the difference between our approach and the strategic delegation approach, which does of course require heterogeneous agents. We extend our approach to encompass heterogeneous agents later in the paper.

A number of assumptions are usually made in formal proofs of the theorem. These are that all voters are identical, in the sense that their probabilities of making the correct choice are the same, they vote independently, they vote sincerely (i.e. not strategically), they can acquire the relevant information costlessly, and voting is costless. Of course these are sufficient conditions; it is possible to relax the assumptions in numerous ways and there are many papers which have investigated whether the theorem holds under less restrictive conditions. Examples are Ladha (1992), who generalizes the theorem to correlated votes; Ben-Yashar and Zahavi (2011), who extend the theorem to include some uninformed voters; Austen-Smith and Banks (1996), and Feddersen and Pesendorfer (1998), who consider strategic voting; and Feddersen and Pesendorfer (1997) and Koriyama and Szentes (2009), who consider the implications of costly information. It seems that the theorem holds under less restrictive conditions than is often thought to be the case, although it is also possible to find counterexamples where extending the number of voters will not improve decision making. A simple example would be where there is initially a single voter, a dictator, who has a probability of 0.9 of making a correct decision. If there are two potential voters, each with a probability of making a correct decision of 0.7, then simple calculations establish that it would not be in the interest of the dictator to extend the franchise to these individuals. But were these individuals each to have a probability of 0.8 of making a correct decision, then extending the franchise to these individuals would raise the probability of making the correct decision, so we would expect the franchise to be extended in these circumstances. It is precisely this kind of calculation that the enfranchised need to make in deciding whether to extend the franchise – extending it will improve collective decision making if the newly enfranchised are equally likely to make a correct decision, and may improve collective decision making even if the newly enfranchised are less good decision makers than the enfranchised.

There is a considerable amount of evidence for the prediction of the Jury Theorem that as the number of decision makers increases, the probability of a correct decision also increases. Surowiecki (2004) gives a large number of examples of how and why such effects operate, although he does not mention Condorcet. A famous example is provided by the British scientist Francis Galton who, in the early twentieth century, discovered that the average of guesses of the weight of an ox at a country fair was almost exactly correct.[4] In an intriguing article, List et al. (2009) apply the theorem to the choice of nest site by honey bees.[5]

It might be argued that the Condorcet theorem applies to decisions where there is one 'correct' answer, whereas most political decisions are about distributional issues where there is no such thing as a 'correct' answer. We do not entirely agree; whereas of course almost all political decisions have distributional implications, most also have implications for efficiency, and this is something for which the Condorcet approach is relevant.

In the model to be presented below we assume that a larger number of voters is likely to have more information and thus make 'better' decisions leading to more 'output', hence giving an incentive to extend the franchise. However, if only voters share in incremental output then the more voters there are the smaller is the share per voter, generating an incentive to contract the franchise. The time path of the franchise depends on the interplay between these two fundamental conflicting incentives. We show that if the information aggregation mechanism is increasing and concave in the number of voters, and when there are no costs to acquiring information, then the dynamics of the franchise follow one of two possible patterns: either the franchise

[4]This is of course not the Condorcet framework, where there is just a binary choice. However, the logic is exactly the same – if each individual has to make a guess about a magnitude and does have some private information, and guesses are independent, then the more people who guess, the more accurate the average of their guesses is likely to be.

[5]However, it has been pointed out to us that Landa (1986) is the original treatment of this issue.

is monotonically increasing to a steady state characterized by universal suffrage, or there is a critical franchise level below which for all initial conditions the system is in a stopping state, implying rule by a small elite or dictatorship. Above this critical franchise level the system again converges to universal suffrage.

As far as we are aware, this is the first attempt to apply the ‘information aggregation’ approach to explaining franchise extension; we believe it does generate a number of results of interest and is not inconsistent with the historical pattern of franchise extension. One franchise extension decision to which we believe this approach is particularly relevant is the decision to extend votes to women. This is something which existing models of franchise extension find extremely difficult to explain, but which can, we argue, be plausibly explained by a suitable extension of our model to include two types of agent, men and women. We accordingly extend our model to incorporate this type of heterogeneity and to argue that it does indeed provide a plausible explanation of female enfranchisement.

The model we have developed is one of franchise extension and growth, and it is interesting to apply it to exploring patterns of growth and development. Even though agents may be homogeneous *ex ante,* they will be heterogeneous *ex post,* in the sense that some may be enfranchised and others disenfranchised, and under our assumptions the enfranchised are better off than the disenfranchised. We are hence able to explore the development of both inequality and growth over time predicted by the model, and, in particular, analyse whether the model can predict a Kuznets curve.

One possible objection to our approach is that the underlying mechanism, based on the additional informational benefits provided by extending the franchise, is too weak to explain the facts of enfranchisement and growth. We are not convinced by this argument – surely the strength of the mechanism is an empirical question but this is something on which there is so far no empirical evidence. We discuss the plausibility of the mechanism further in Section VI, and postulate a number of ways, consistent with the overall approach, whereby a franchise extension may raise productivity.

So, to summarize, the paper is structured as follows: Section II presents the baseline model we use and Section III gives our basic results. Section IV shows how the approach can be extended to explain female enfranchisement and the Kuznets curve, while Section V discusses relevant historical evidence for these explanations. Section VI discusses the relationship between franchise extension and productivity in more detail, whilst Section VII concludes.

II. BASELINE MODEL

The structure of our basic model involves a simple version of an endogenous growth model (Romer, 1990) combined with a public decision-making process based on voting by enfranchised individuals. We assume the economy to be populated by a continuous interval of individuals of length n. We also assume a discrete-time, infinite-horizon framework with initial period 0. The infinitely-lived individuals are identical in every regard except for whether or not they are enfranchised. Denote $v(t) \in [0, n]$ as the number (mass) of members of the population enfranchised at time t.

II.1 Economics

We assume that in each period there are available a number $c(t)$ of potentially productive choices with $c(0) > 0$ (otherwise the economy would never get going). If a potentially productive choice is realized, termed hereafter a successful choice, it yields a single unit of output. We denote by $s(t)$ the sum of all successful choices at t. Each successful choice generates $\lambda s(t)$ potential choices for the next period, so $c(t+1) = \lambda s(t)$. Suppose, for example, that one element of $c(t)$ is the choice of where to construct a new road; if construction takes place in the 'correct' place this facilitates commerce and hence output increases, and also leads to future decisions (such as the construction of connecting routes). We assume $\lambda \geq 1$ so that there are non-decreasing returns to scale in successful choices. Total output in a period is then just the sum of successful choices. We assume that all individuals receive a fixed output payoff per period of m and that the enfranchised individuals share equally in the incremental output, $s(t)$. For simplicity we then normalize the fixed payoff per individual to zero (we shall relax this assumption appropriately later). The income in period t of an enfranchised individual, $\pi(t)$, is thus

$$\pi(t) = \frac{s(t)}{v(t)} \tag{1}$$

II.2 Political decision making

In each period all enfranchised individuals have to decide how large they would like the franchise to be in the next period, and what to do with each potentially productive choice. For each productive choice we assume the decision is binomial – that is, there is a correct decision and an incorrect one. We assume that the proportion of productive choices over which decisions are correct is determined by the number of voters involved and is given by $f(v(t))$. We therefore have $s(t) = f(v(t))c(t)$. We assume $f(.)$ is increasing, concave, that $f(.) \in [0, 1]$, $\lim_{v \to 0} f(v(t)) \to 0$ and $\lim_{v \to 0} \frac{\partial f(v(t))}{\partial v(t)} \to +\infty$. By assuming $f(.)$ is increasing we are essentially assuming that the Condorcet Jury Theorem holds. There are a number of interpretations possible for this structure and we shall provide some examples later. The franchise extension decision is a little complex, but since voters are identical all voting rules maximize the expected payoff of a representative voter who may conveniently be thought of as median. A complication arises when there is a possibility of the franchise declining; since voters are all the same we assume that in these circumstances they each face an equal probability of being disenfranchised. However if the franchise is extended, each currently enfranchised individual is guaranteed to be enfranchised in the next period. The payoff to enfranchised voters may thus be written

$$V(v(t), c(t)) = \left[\frac{f(v(t))}{v(t)}\right] c(t) + \delta \min\left[v(t+1)/v(t), 1\right] V(v(t+1), c(t+1)) \tag{2}$$

where δ is the discount factor. This equation states that the value to a voter of being in the state characterized by $v(t)$ voters and $c(t)$ potentially productive choices equals the flow benefits of being in that state, namely consumption received in that period, plus the discounted present value of the next period's expected payoff.

III. BASIC RESULTS

III.1 Steady-state solutions

We begin our analysis by supposing that there exists a steady-state solution in which a constant proportion of the population is enfranchised, that is $v(t) = \overline{v} \forall t \geq t_0$. Our main concern here is to characterize the circumstances under which the system achieves universal suffrage, $\overline{v} \geq n$, and hence under which it does not, $\overline{v} < n$. To analyse this, we consider the choice of the franchise for period t_0, assuming it is expected that the franchise will be at its steady state level in all future periods. By repeated substitution in (2) we can show that the payoff of an enfranchised voter at time t_0 when the franchise is $v(t_0)$ and is expected to be $\overline{v}$ in all periods $t > t_0$ is

$$V(v(t_0), c(t_0)) = \left[\frac{f(v(t_0))}{v(t_0)}\right] c(t_0) + \delta\lambda \left[\frac{f(\overline{v})}{\overline{v}}\right] [c(t_0) f(\overline{v})] + \delta^2\lambda^2 \left[\frac{f(\overline{v})}{\overline{v}}\right] [f(\overline{v})] [c(t_0) f(\overline{v})] + \cdots \quad (3)$$

In deriving this we assume that the franchise is never expected to decline. We show later in the paper that this is indeed the case. For $\overline{v}$ to be a steady state we require that given $v(t) = \overline{v}$ $\forall t \geq t_0 + 1$ then the enfranchised voter would also choose $v(t_0) = \overline{v}$. The tradeoff in choosing the optimal franchise for period t_0 is as follows: an increase in the period t_0 franchise increases current income because more potentially productive choices are realized. But this also increases future incomes because if more potentially productive choices are realized now, this means there are more potentially productive choices in the future. The cost of an increase in the franchise (to the decision maker who expects to be enfranchised in period t_0) is that this income will be shared more widely. Performing the optimization problem and rearranging (see the Appendix) and defining $\varepsilon(v) \equiv \frac{\partial f(v)}{\partial v} \frac{v}{f(v)}$ as the efficiency elasticity of the information aggregation (voting) process, we have as the necessary condition for an interior optimum:

$$\varepsilon(\overline{v}) \left[\frac{1}{1 - \delta\lambda f(\overline{v})}\right] = 1. \quad (4)$$

We may immediately state:

Proposition 1. If $\varepsilon(n)[\frac{1}{1-\delta\lambda f(n)}] \geq 1$ *a steady state exists with universal suffrage, whereas if* $\varepsilon(\overline{v})[\frac{1}{1-\delta\lambda f(\overline{v})}] = 1$ *for* $0 \leq \overline{v} < n$ *a steady state exists with less than universal suffrage, and if* $\varepsilon(0)[\frac{1}{1-\delta\lambda f(0)}] \leq 1$ *a steady state exists that would involve dictatorship.*

The proofs of this and all subsequent propositions are relegated to the Appendix. The steady state occurs where the marginal contribution to output achieved by superior decision making as a consequence of adding the last enfranchised voter just equals the marginal loss in output to enfranchised voters from splitting output between more individuals. We may also state:

Proposition 2. The steady-state franchise tends to be larger: (1) the less the future is discounted, $\frac{d\overline{v}}{d\delta} > 0$*, (2) the greater are the returns to scale in successful choices,* $\frac{d\overline{v}}{d\lambda} > 0$.

This is intuitive. The greater is δ the less the future is discounted. Adding an extra voter in the current period increases the number of successful choices possible both in that and in all future periods, and it also increases the number of individuals sharing in output for all periods until the steady state is achieved. Since an increase in δ increases the weight placed on both future benefits and costs, and the stream of future benefits increases indefinitely whereas the increases in costs are constant at the steady state, it follows that this tends to increase the steady-state size of the franchise.

The greater are the increasing returns to successful choices, λ, the higher the benefits from improving decision making at any time and hence the greater the steady-state level of enfranchisement.

We now wish to explore how improvements in the efficiency of the information aggregation mechanism affect the steady-state level of the franchise. To facilitate this we rewrite the mechanism as $f(\overline{v}, \alpha)$ where α is a parameter that may be used to explore the effects of exogenous changes in the intercept or slope of the information aggregation function. We can derive the following results:

Proposition 3. (i) An increase in the efficiency of the information aggregation process of the form $f_\alpha > 0$ and $f_{\bar{v}\alpha} = 0\ \forall \bar{v}$ increases (decreases) the steady-state franchise level if $\varepsilon(\bar{v}) < (>)1/2$; (ii) an increase in the efficiency of the information aggregation process of the form $f_\alpha = 0$ and $f_{\bar{v}\alpha} > 0\ \forall \bar{v}$ increases the steady-state franchise level.

The first change raises the intercept but not the slope of the function; the second does the reverse. The intuition here is straightforward: for the case $f_\alpha = 0$ and $f_{\bar{v}\alpha} > 0$ an increase in α unambiguously raises the efficiency elasticity of the information aggregation mechanism, since the latter can be written as the marginal product of the information aggregation function divided by the average product, and the change we are considering raises the marginal product without changing the average product. This tends to raise the franchise. For the case $f_\alpha > 0$ and $f_{\bar{v}\alpha} = 0$, the elasticity of the information aggregation process falls unambiguously with an increase in α (the average product of the information aggregation function increases whilst the marginal product stays unchanged) tending to reduce the steady-state franchise; however the future benefits from a correct decision today (reflected in the term $\frac{1}{1-\delta\lambda f(\bar{v})}$) increase with $f_\alpha > 0$. The former effect is greater the lower the initial value of the elasticity of the information aggregation function, and the value of the elasticity at which the two effects cancel is shown in the Appendix to be 1/2.

III.2 Dynamics

Inspection of equation (2) reveals that the enfranchised agents' optimization problem is non-standard. Extensions and contractions of the franchise are valued using different forms of the payoff function. This follows from the plausible assumption that when the franchise is extended all current voters are necessarily enfranchised in the next period, whereas if the franchise contracts each current voter has an equal chance of being disenfranchised. Fortunately we can simplify matters:

Proposition 4. The franchise is everywhere non-decreasing, $v(t+1) \not< v(t)\ \forall t$.

Proposition 4 follows from a simple observation, that if $v(t+1) < v(t)$ then the probability that an enfranchised agent in t is also enfranchised in $t+1$ is simply $\frac{v(t+1)}{v(t)}$, while the share of output enjoyed by the voter if enfranchised in $t+1$ is $(\frac{f(v(t+1))}{v(t+1)})c(t+1)$ giving an expected value of $(\frac{v(t+1)}{v(t)})(\frac{f(v(t+1))}{v(t+1)})c(t+1) = (\frac{f(v(t+1))}{v(t)})c(t+1)$ which is increasing in $v(t+1)$. Hence whenever $v(t+1) < v(t)$ the marginal value of an increase in $v(t+1)$ is strictly positive, so that it is impossible for an optimal path to involve $v(t+1) < v(t)$. In other words, if the number of voters is expected to decrease, each current voter expects to obtain the same fraction of the 'cake', but since the size of the cake declines each voter expects to be made worse off by a franchise contraction and so will not vote for it.

The significance of the result is not so much that franchise contraction cannot occur, but that a model with homogeneous agents cannot explain franchise contraction (whereas it can explain many episodes of franchise expansion). It follows that to explain franchise contractions, we need either an entirely different approach or to take explicit account of agent heterogeneity. It might seem that this property of the model that the franchise cannot decline is undesirable. Situations where the franchise has indeed contracted are not unknown, particularly in South America. However, several observations might be pertinent. First, franchise contractions in South America have typically been the result of coups or revolutions, events that could be treated as exogenous shocks to the model presented above. Second, some countries such as Britain, Canada, and New Zealand have not experienced franchise contractions and have behaved in a manner not inconsistent with this theoretical structure. Finally, note that the theoretical model is explicitly constructed to demonstrate that an information-based story completely devoid of any incentives for strategic delegation can do a good job of explaining many seemingly paradoxical episodes of voluntary franchise extension. To this end agents in the model are assumed to be identical, leading to the natural assumption that were the franchise to decline each agent faces an equal chance of being disenfranchised. Were there to be some heterogeneity in the model, franchise reductions might arise. Despite the allure of this, we continue for the moment to explore what can be learned from a model populated by homogeneous agents.

Given Proposition 4 we need consider the enfranchised agent's payoff function only in the form

$$\begin{aligned} V(v(t), c(t)) &\equiv \left[\frac{f(v(t))}{v(t)}\right] c(t) + \delta V(v(t+1), c(t+1)) \\ &= \left[\frac{f(v(t))}{v(t)}\right] c(t) + \delta\lambda \left[\frac{f(v(t+1))}{v(t+1)}\right] [c(t) f(v(t))] \\ &\quad + \delta^2\lambda^2 \left[\frac{f(v(t+2))}{v(t+2)}\right] [f(v(t+1))] [c(t) f(v(t))] + \cdots \\ &= c(t) \sum_{i=0}^{\infty} \left[\frac{(\delta\lambda)^i}{v(t+i)}\right] \left[\prod_{j=0}^{i} f(v(t+j))\right] \end{aligned} \tag{5}$$

The first-order condition for an interior optimum then requires

$$\begin{aligned} \frac{\partial V(v(t), c(t))}{\partial v(t)} &= \left[\frac{\left(\frac{\partial f(v(t))}{\partial v(t)}\right) v(t) - f(v(t))}{v(t)^2}\right] c(t) + \delta\lambda \left[\frac{f(v(t+1))}{v(t+1)}\right] \left(\frac{\partial f(v(t))}{\partial v(t)}\right) c(t) \\ &\quad + \delta^2\lambda^2 \left[\frac{f(v(t+1)) f(v(t+2))}{v(t+2)}\right] \left(\frac{\partial f(v(t))}{\partial v(t)}\right) c(t) + \cdots = 0 \end{aligned} \tag{6}$$

and the second-order condition can be stated as

$$\begin{aligned} \frac{\partial^2 V(v(t), c(t))}{\partial v(t)^2} &= \left(\frac{\partial^2 f(v(t))}{\partial v(t)^2}\right) c(t) \left[\frac{1}{c(t)v(t)} + \delta\lambda \left[\frac{f(v(t+1))}{v(t+1)}\right]\right. \\ &\quad \left. + \delta^2\lambda^2 \left[\frac{f(v(t+1)) f(v(t+2))}{v(t+2)}\right] + \cdots\right] - 2 \left[\frac{\left(\frac{\partial f(v(t))}{\partial v(t)}\right) - \frac{f(v(t))}{v(t)}}{v(t)^2}\right] < 0 \end{aligned} \tag{7}$$

It should be clear that $\varepsilon^v(t) < 1$ (we define $\varepsilon^v(t) \equiv \varepsilon(v(t))$ is a necessary condition for an interior solution; otherwise the first term on the right-hand side of (6) would not be negative and the whole expression could hence not be (non-trivially) zero. This is ensured by the properties of $f(v(t))$, but for this interior solution to be a maximum we also require the information aggregation function to be sufficiently concave, that is $\frac{\partial^2 f(v(t))}{\partial v(t)^2}$ be absolutely sufficiently large, such that $\frac{\partial^2 V(v(t),c(t))}{\partial v(t)^2} < 0$. We shall maintain this assumption hereafter. Manipulation of the FOC (see the Appendix) yields the expression that characterizes the dynamics of enfranchisement

$$\frac{v(t+1)}{v(t)} = \delta\lambda f(v(t+1)) \left[\frac{\varepsilon^v(t)}{1 - \varepsilon^v(t)}\right] \left[\frac{1}{\varepsilon^v(t+1)}\right] \tag{8}$$

While it might appear that this equation implies complicated dynamics, this is not in fact the case. Since this analysis is only legitimate if $v(t+1) - v(t) \geq 0$, if it appears that the franchise

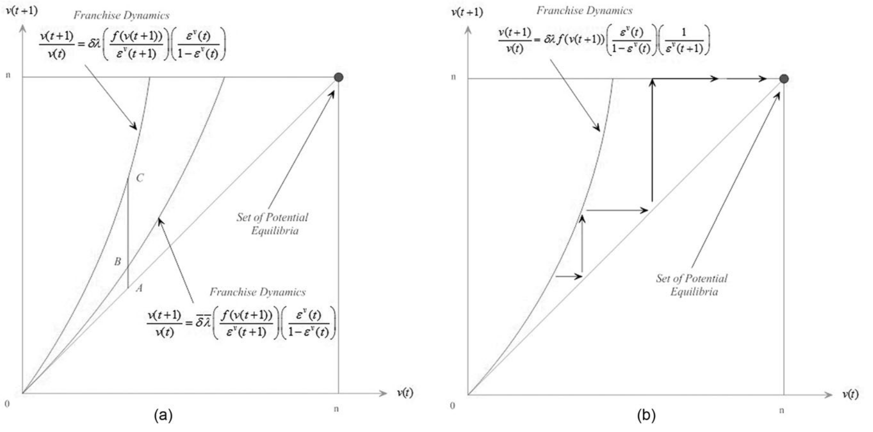

Fig. 1. (a) Convergence to universal suffrage (b) Small elite or universal suffrage.

must decline then we know that Proposition 4 applies and the dynamics must reach a stopping state.

III.2.1 Possible time paths for enfranchisement. We know from Proposition 4 that the franchise cannot decline, hence the dynamics must be characterized by stopping states, franchise expansions, or some combination of the two. We have:

Proposition 5. (i) $1 < \delta\lambda f(0)$ *is a sufficient condition for all interior franchise expansion paths to converge monotonically to universal suffrage whereas (ii) if the solution to the agent's optimization problem is interior and* $1 \geq \delta\lambda f(0)$ *then all* $v(t) \in [0, v^*]$ *are stopping states while* $v(t) \in (v^*, n]$ *is a region of monotonic convergence to universal suffrage.*

These time paths are illustrated in Figure 1.

The intuition for part (i) of this proposition may be teased out of expression (8); since we know that along any interior franchise expansion path $\varepsilon^v(t) \in (0, 1)$ and $\frac{\partial \varepsilon^v(t)}{\partial v(t)} < 0$, then $\frac{\varepsilon^v(t)}{\varepsilon^v(t+1)} > 1$ and $\frac{1}{1-\varepsilon^v(t)} > 1$ hence $1 < \delta\lambda f(1)$ is sufficient for the result. If correct decisions tomorrow are sufficiently productive then it is always worthwhile extending the franchise. Part (ii) follows from the recognition that at low levels of the franchise the number of correct decisions $f(v(t+1))$ may be very low so that despite the increasing returns to correct decisions ($\lambda > 1$), the level of output may tend to decline, leaving the economy in a stopping state. However for higher levels of the franchise, enough successful decisions are made to trigger output growth which increases the marginal returns to enfranchising more voters, leading to further enfranchisements.

These results reflect the fundamental tension in the model. Key to the behaviour of the dynamics is the efficiency elasticity of the information aggregation mechanism, ε. An increment to the franchise in any period raises output proportionately to the marginal improvement in the quality of decision making ($\partial f(v)/\partial v$), but results in more individuals sharing in output, the cost of which is approximately proportional to the average quality of decision making ($f(v)/v$). The efficiency elasticity of the information aggregation mechanism is precisely the ratio of marginal to average. So if the benefit of another voter is larger relative to the cost, the greater is the elasticity, the more individuals tend to be enfranchised. How rapidly the marginal

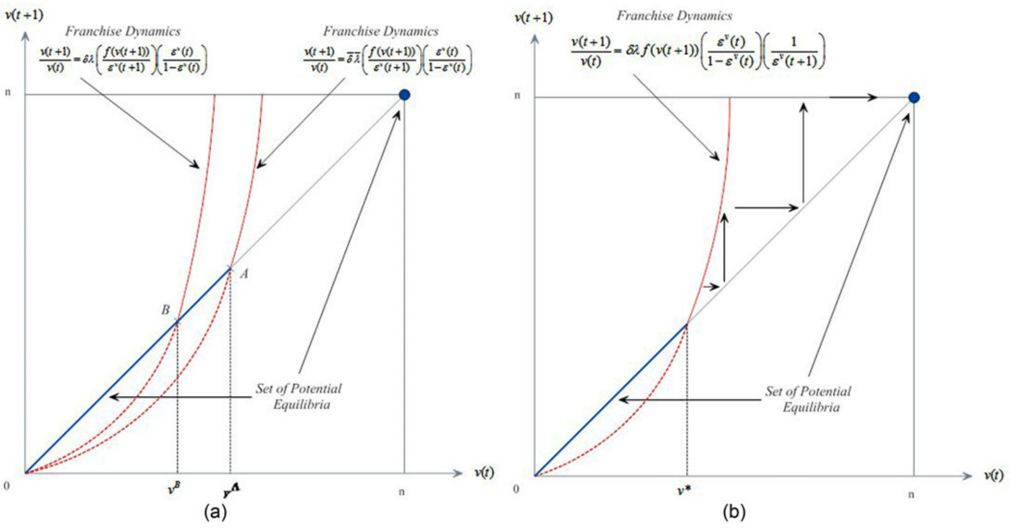

Fig. 2. (a) Speed of enfranchisement: monotonic case (b) Speed of enfranchisement: case with stopping states.

declines relative to the average thus determines whether the system achieves full or limited enfranchisement.[6]

III.2.2 Comparative dynamics. The results in the preceding section indicate that an information based story can explain why the time path of the franchise may lead to universal suffrage in some economies but remain stuck in oligarchy in others. It remains to ask both the question of how rapidly universal suffrage will be achieved in those instances where it characterizes the steady state, and what the characteristics of the time path look like. Changes in the speed of enfranchisement involve shifts in the difference equation (8); furthermore, it is clear that if universal suffrage is achieved more rapidly, this involves fewer larger incremental increases. Formally we may show:

Proposition 6. The effects of changes in δ and λ

(i) *As δ and λ increase along time paths that involve franchise extensions;*
   a. *A necessary and sufficient condition for the rate of enfranchisement to be greater in the sense $\left.\frac{dv(t+1)}{d(\delta\lambda)}\right|_{v(t)} > 0$ is $\frac{\partial f(v(t+1))}{\partial v(t+1)} v(t+1)[1- \varepsilon^{v}(t+1)] + v(t+1)\frac{\partial \varepsilon^{v}(t+1)}{\partial v(t+1)} > 0$,*
   b. *A sufficient condition for the rate of enfranchisement to be higher is $\frac{\partial \varepsilon^{v}(t+1)}{\partial v(t+1)} > 0$,*
   c. *A sufficient condition for the rate of enfranchisement to be smaller is that $\varepsilon^{v}(t+1)$ be convex in $v(t+1)$,*

(ii) *For time paths involving stopping states then the conditions in part (i) for an increase in the rate of franchise expansion are also conditions for a decrease in the measure of the set of stopping states.*

Figure 2 illustrates these conclusions.

[6]It might be thought that there could be a potential third dynamic path type where the difference equation cuts the $v(t) = v(t+1)$ line from above. However, this can be ruled out by differentiating (8) with respect to $v(t)$ and $v(t+1)$ and evaluating the resultant expression at $v(t) = v(t+1)$ using $\frac{\partial \varepsilon}{\partial v} < 0$ and $\varepsilon \in (0, 1)$.

If the conditions for an upward shift in the difference equation (8) are satisfied, then for a monotonically convergent time path as illustrated in Figure 2a, the implications of an increase in δ or λ involve a greater increment in the franchise from period t to period $t+1$ for any given initial franchise level as shown by C–A compared to B–A. For time paths involving stopping states, we see, as illustrated in Figure 2b, that under the same conditions the set of stopping states changes from v^A to v^B.

IV. APPLICATIONS TO FEMALE ENFRANCHISEMENT AND THE TIME PATH OF INEQUALITY

In this section we discuss how our approach can be extended to shed light on two phenomena of importance: the history of female enfranchisement, and the dynamics of inequality.

An inspection of Figure 2b suggests that it is possible that an exogenous shift in the difference equation describing franchise dynamics, equation (8), may cause the system in a given state to jump from a regime of stopping states to one of monotonic franchise increases. This has several possible applications, perhaps the most interesting of which is the extension of the vote to women. It is very noticeable that in many countries the franchise was extended to women around the end of World Wars I or II. Our analysis offers two possible explanations for this. First, we might argue that pre-war these economies were stuck in stopping states with about half the adult population enfranchised (i.e. $v(t) \approx n/2$). It is possible that the war triggered an increase in δ, reflecting greater patience in the immediate post-war period, or an increase in λ as a consequence of wartime damage leading to more potential choices. Both of these effects could move the economy from a stopping state to a region of monotonic franchise expansions. These explanations are unconvincing as they beg the question of why the first $v(t) \approx n/2$ voters enfranchised were male.

A more persuasive hypothesis is that male and female voters received differential information. In the pre-war period most males obtained employment outside the home while a large percentage of females were employed within the home. However, the wars removed a large number of potential male voters from the economy via military service and reallocated their domestic economic roles to females. It seems reasonable to argue that the information set of female voters 'improved' while the information set of males 'deteriorated'. To examine this idea we redefine our information aggregation function in terms of efficiency units of votes, $\alpha v(t)$, and indicate male (female) voters efficiencies by α^M and α^F respectively. It is straightforward to show that for interior franchise extension paths the dynamics of the system obey a difference equation of the form

$$\frac{v(t+1)}{v(t)} = \delta\lambda f(\alpha v(t+1)) \left[\frac{\varepsilon^{\alpha v}(t)}{1-\varepsilon^{\alpha v}(t)}\right] \left[\frac{1}{\varepsilon^{\alpha v}(t+1)}\right] \tag{9}$$

where $\varepsilon^{\alpha v}(t) \equiv \frac{\partial f(\alpha v(t))}{\partial(\alpha v(t))}\frac{\alpha v(t)}{f(\alpha v(t))}$. An important property of the dynamics is given by

Proposition 7. The measure of the set of stopping states is decreasing in α^M and α^F.

This establishes that if the efficiency of voting increases (decreases) then the graph of the difference equation shifts leftwards (rightwards) in phase space (see Figure 3).

We now employ this proposition together with a pair of plausible assumptions to explain the observed patterns of male and female enfranchisement. First, consistent with our observations of male and female pre-war employment experiences we assume that males are better informed for this period, that is, $\alpha^M > \alpha^F$. This, as we shall see, is a necessary condition for males to be enfranchised first. Post-war our story involves $\alpha^F < \alpha^{F'}$ and $\alpha^{M'} \leq \alpha^M$ where a prime indicates

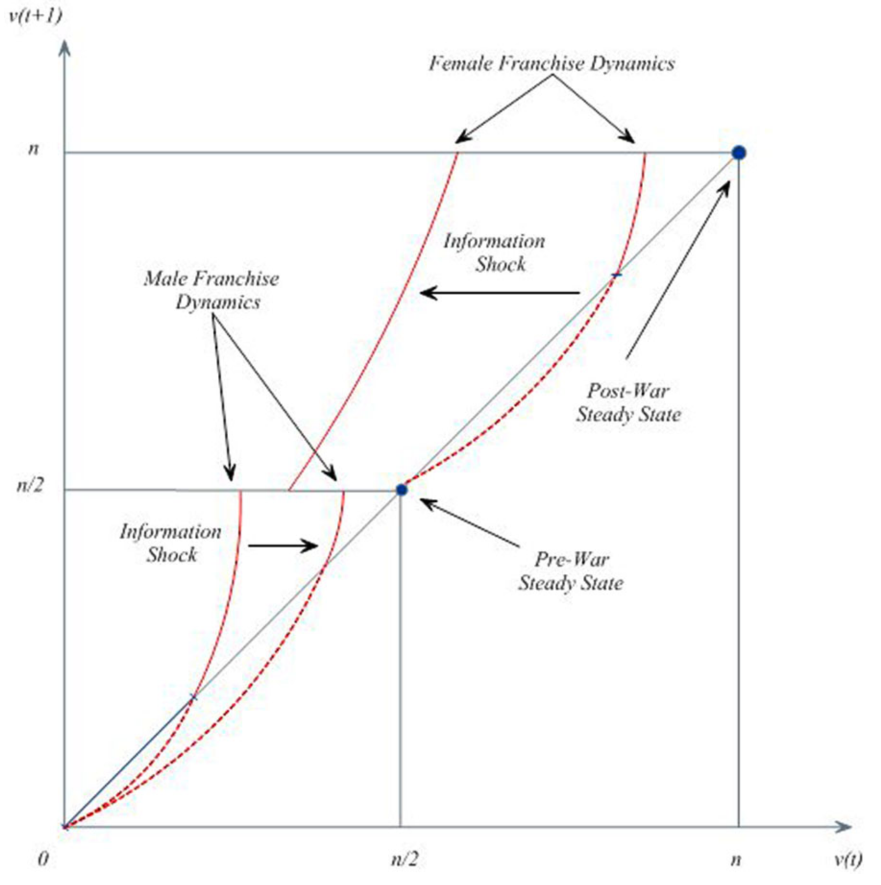

Fig. 3. Female enfranchisement.

the post-war parameter value. Females become better informed on the relevant margins due to their wartime experiences whereas males do not. Figure 3 shows the dynamics of female enfranchisement.

The story is now quite straightforward: prior to the war males enjoyed an informational advantage and full suffrage.[7] However, female enfranchisement was stuck in a stopping state involving compete disenfranchisement. The informational shocks associated with the war shifted the ‘female’ section of the difference equation leftward but did not do the same for the ‘male’ section. The post-war steady state now involves full female enfranchisement, because of the improvement in the efficiency of female votes, and continued full male enfranchisement because the franchise cannot decline.[8]

In the simple model developed above there are two income classes, the enfranchised and the unenfranchised. To facilitate an examination of the time path of inequality, let each agent irrespective of whether they are enfranchised receive a non-zero constant subsistence wage m in addition to any claims on the incremental output $s(t)$. The agents’ payoff functions only differ by the additive constant $\frac{m}{1-\delta}$ implying that in the dynamics of enfranchisement continue

[7]In reality male suffrage was very high but not 100 percent.
[8]Note that a decline in the quality of male information is not necessary for this result.

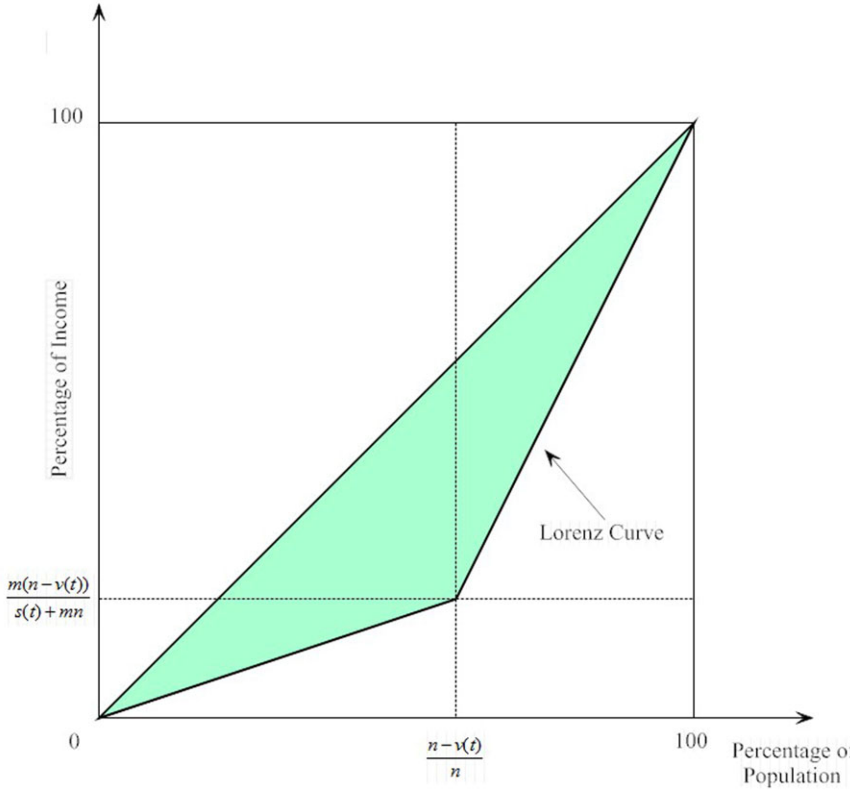

Fig. 4. Lorenz curve and Gini coefficient.

to be governed by (8). As our measure of inequality we adopt the Gini coefficient which in this framework is given by the following expression:

$$G(v(t)) = 1 - 2\left[\left(\frac{v(t)}{n}\right)\left(\frac{m(n-v(t))}{s(t)+mn}\right) + \left(\frac{1}{2}\right)\left(\frac{m(n-v(t))}{s(t)+mn}\right)\left(\frac{n-v(t)}{n}\right) + \left(\frac{1}{2}\right)\left(1 - \frac{m(n-v(t))}{s(t)+mn}\right)\left(\frac{v(t)}{n}\right)\right]$$
$$= \frac{s(t)}{n}\left(\frac{n-v(t)}{s(t)+mn}\right) = \frac{f(v(t))\,c(t)}{n}\left(\frac{n-v(t)}{f(v(t))\,c(t)+mn}\right) \quad (10)$$

This corresponds to the shaded area between the Lorenz curve and the complete equality line in Figure 4.

We may now study the behaviour of the Gini coefficient over time. We know that there are two possible types of time path the franchise may follow – it either converges monotonically to universal suffrage or is stuck in a stopping state. Applying these results to expression (10) we may derive:

Proposition 8. (i) If the economy is in a stopping state then inequality is increasing (decreasing) as $\lambda f(\overline{v}) > (<) 1$; *(ii) If the economy is monotonically convergent to universal*

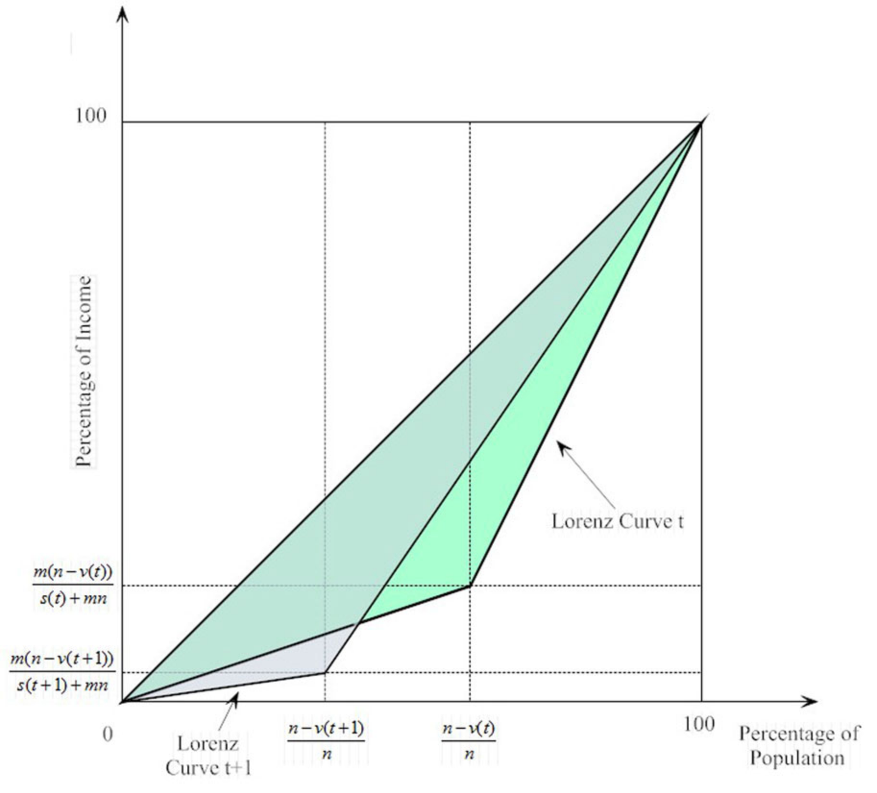

Fig. 5. Gini coefficient dynamics.

suffrage then the dynamic system satisfies the following necessary conditions for the existence of a Kuznets curve: $\left.\frac{dG(v(t))}{dv(t)}\right|_{v(t)=0} > 0$ *and* $\left.\frac{dG(v(t))}{dv(t)}\right|_{v(t)=n} < 0$.

The first part of Proposition 8 is straightforward; if the number of enfranchised agents is constant then so too is the quality of decision making. The enfranchised get richer, and inequality increases if the returns to scale parameter, λ, is large enough to overcome the poor quality of decisions made by the small decision-making group. If not the enfranchised become poorer, output deteriorates and inequality falls. In the cases where the franchise is increasing, as in the second part of Proposition 8, matters are more complicated. We illustrate the dynamics of inequality for this case in Figure 5.

Along a path of franchise extensions there are two effects on inequality: more individuals join the group sharing in incremental output and this tends to reduce inequality; however, incremental output is growing over time, increasing the gap between the incomes of the enfranchised and unenfranchised groups tending to increase inequality. The second part of Proposition 8 follows from considering what must happen to these two effects as the franchise approaches its limits. On the one hand, as $v(t) \to 0$ the dominant influence is the growth effect since $\frac{df(v(t))}{dv(t)}|_{v(t)\to 0} \to \infty$, whereas on the other, as $v(t) \to n$ inequality must fall as the number of individuals in the poor unenfranchised group approaches zero.

V. HISTORICAL EVIDENCE

As far as female enfranchisement is concerned, the following seem the relevant facts to be explained: in virtually all countries that gave women the vote, this did not happen before all their male counterparts had received the vote.[9] Furthermore, women were often enfranchised en masse, and in many countries the expansion of the franchise to women followed one of the two world wars. For example, in 1918 the vote was extended to women over the age of 30 and subject to some qualifications in the UK; in that year it was also extended to women in Austria, Canada, and Poland. Women received the vote in Germany and the Netherlands in 1919, in the USA in 1920, in Sweden in 1921, and in Ireland in 1922.[10] Around the end of World War II, women received the vote in Japan in 1945, in France and Italy in 1946, in Belgium in 1948, and in Greece in 1952. Empirical analysis presented by Hicks (2013) shows that ‘participation in external conflict by a nation previously lacking female suffrage more than doubles the probability of female enfranchisement in the immediate post-conflict period, even after controlling for other commonly theorized determinants of suffrage expansion’ (pp. 60–61) We would argue that an information-based explanation of franchise extensions fits well with this evidence. The world wars caused massive redistributions of tasks between males and females; many females entered the workforce and worked in areas of the economy in which they were previously significantly underrepresented. It seems quite probable that the information content contained in female votes jumped discontinuously as a consequence of this upheaval, and occurred at the same time as the information content of the male votes fell. Casually it makes sense then for females to be enfranchised and this is precisely what we have shown formally. This explanation can be strengthened by noting that many countries emerged from the wars with a large surplus of women. Moreover, the men who had been killed or wounded during the wars were predominantly in the 18–40 age group, and would have been economically active after the war. So there may have been many key positions which men were not available to fill, and which hence might be filled by women. But also the surplus of women meant that many women would be unable to find husbands, and hence much more likely to work for this reason. (The difference between the labour force participation rates of married and unmarried women in the early twentieth century was huge – see, for example, Costa, 2000, p. 106.) So both supply and demand considerations implied that higher female force participation was anticipated to continue after the war, with associated informational benefits from enfranchisement. The benefits are raised further if the enfranchisement was thought to enable more women to reach positions of power and influence.

There is some economics literature on female enfranchisement, as well as an extensive literature in political science and history. However, in the context of female enfranchisement, the basic question which motivated the franchise extension literature in economics, namely why an elite should choose to dilute its power, seems to have scarcely been asked, let alone answered. It would seem possible to think of the following possible explanations for a decision by men to grant the vote to women: (1) Men realized the inherent injustice of denying women the vote and extended the vote to women out of altruism; (2) Men gave women the vote because of the considerable disruption caused by the suffragette campaign (the ‘revolutionary

[9] Sometimes, as in the UK in 1918, universal male enfranchisement was completed at the same time as votes were first extended to women.

[10] We use the dates for female enfranchisement given in appendix A of Hicks (2013). We do not discuss the issues involved in assigning a date to female enfranchisement, except to note that the dates given are generally the dates when national legislation gave women the vote in these countries. Often particular states or regions of some of these countries granted women the vote earlier, as in the USA where twelve US states had given women the vote before the nineteenth amendment to the constitution was ratified in 1920.

threat' explanation of franchise extension); or (3) men gave women the vote because a significant fraction of the electorate found it in its interests to do so (the 'split elite' explanation of franchise extension). The first explanation is extremely difficult to reconcile with the fact that there were waves of female enfranchisement in the aftermath of both world wars. (Did men suddenly become more altruistic after the wars?) We are unaware of any scholar who has propounded this as the main explanation for female enfranchisement, although it may have motivated some of those who supported such reforms. The second explanation may be slightly more plausible. However, it is also difficult to reconcile with the timing evidence, and it has been rejected by historians, at least as far as the UK is concerned (e.g. Smith, 1998, p. 42), who argue that suffragette violence may have been counterproductive by antagonizing public opinion. The third explanation is perhaps related to Doepke and Tertilt's explanation for women's liberation: see Doepke and Tertilt (2009).[11] Their idea is that men gave women more rights in the nineteenth century because they are altruistic towards their daughters; if their daughters have more rights, they have more bargaining power and benefit in the marriage market; they may also invest more in human capital. We are not entirely persuaded by this argument – if women's bargaining power increases, this may benefit men with daughters, but might it also harm men with sons? Nevertheless, this is not an explanation for women's suffrage but for the granting of some legal rights to married women, which took place before women received the vote. We are hence left with no plausible alternative to our explanation for franchise extension to women, which is that men thought they would benefit from the participation of women in elections because it would mean better decision making.[12]

One paper that seeks to explain women's enfranchisement is Bertocchi (2011), which presents a model in which there is a cost to male voters of enfranchising women, but also a benefit. The costs stem from the fact that women's suffrage means that public spending is higher (meaning higher taxes) and the type of public spending is different (there is more spending on public goods preferred by women). These costs would fall with economic development, hence explaining women's enfranchisement over time. The benefit of enfranchising women (described as the 'societal cost of women's disenfranchisement'), which of course must be positive for enfranchisement to take place, is not really explained (it is said to be 'related to a country's culture'; Bertocchi, 2011, p. 542). Our paper gives a different interpretation of these benefits. They have to do with the informational benefits of enfranchising women. These can quite plausibly change considerably in a short period of time, which would seem unlikely for cultural factors. Our explanation is hence quite compatible with Bertocchi's; while she concentrates on the costs, we analyse the benefits of enfranchisement in more detail.

Another paper that emphasizes the role of wars in explaining enfranchisement is Ticchi and Vindigni (2009). Their explanation for enfranchisement is that the elite may promise the vote to motivate those fighting in a war. However, their explanation is not explicitly designed to explain female enfranchisement; it may explain the male enfranchisement that did take part after the world wars, such as in the United Kingdom where the Representation of the Peoples Act of 1918 removed the remaining property qualifications on men (as well as enfranchising women over 30 subject to a property qualification), but it is not clear how applicable it is to female enfranchisement. But also, there would seem to be a time consistency problem with this

[11] In a somewhat similar vein, Fernandez (2014) considers the extension of property rights to women.

[12] It has been suggested to us that a change in the bargaining position of women within households because of wars may have been responsible for female enfranchisement. But it is not clear in which direction the wars affected womens' bargaining positions. The increased availability of paid work may have increased their bargaining power, whereas the shortage of men may have had the opposite effect. And even if their bargaining position were strengthened, it needs to be explained why this led to men granting women the vote, rather than transferring more resources to them within the household.

explanation of enfranchisement – why would a promise to enfranchise either men or women after the end of the war be credible? It is also not clear whether this explanation is compatible with the fact that some countries that enfranchised women in the aftermath of wars (e.g. Germany in 1919 and Japan in 1945) had lost the wars.

There is evidence that giving women the vote led to effects on both the composition and level of public expenditures, hence confirming our assumption that groups who obtain the vote thereby obtain more resources than those who do not. Lott and Kenny (1999) and Aidt and Dallal (2006) provide evidence for US states and a variety of European countries respectively.

A number of countries and four US states that enfranchised women well before World War I may seem to provide a difficulty for our explanation. The countries and US states are (with the date of enfranchisement given in parentheses): New Zealand (1893), Australia (1902), Finland (1906), Wyoming (1869), Utah (1870), Colorado (1893), and Idaho (1896). However, all these countries and states (except Finland) were ‘frontier societies’ with large surpluses of men and wished to encourage large-scale immigration, particularly of women. By giving women voting rights, they were making themselves more attractive to women, and by making themselves more attractive to women they were of course making themselves more attractive to men.[13] Finland’s enfranchisement of women can perhaps be explained by the fact that in 1906 it was part of the Russian empire and had been hugely affected by the 1905 Russian Revolution, which caused a considerable amount of disruption; it can hence be regarded as the first example of female enfranchisement for which our explanation is plausible.

Sweden’s enfranchisement of women in 1919 can perhaps be similarly explained by the fact that although it had not taken part in World War I, it had experienced considerable disruption and turbulence in this period. On the other hand, Switzerland, which was the last European country to give women the vote, in 1971, was not involved in either of the world wars; indeed, the most recent conflict to take place on its soil was a fairly minor civil war in 1847.

Female enfranchisement in some Latin American countries (such as Argentina in 1951 and Mexico in 1953) may also be difficult to reconcile with our explanation, as they had only limited involvement in World War II, although they did have a fairly turbulent history in the decades before the enfranchisement decision. However, we would readily concede that there are others factors which may be responsible for female enfranchisement over and above those we have considered in this paper – probably the level of economic development and spillover effects from neighbouring countries are quite relevant as well, and these factors may have been important for these countries.[14]

As the analysis above demonstrates, an information aggregation story is able to shed light on the relationship between franchise extensions, growth, and inequality and is able to generate a political economy explanation of the Kuznets curve, whereby inequality first rises, and then falls, over the period of economic growth. The existence or absence of a Kuznets curve has received considerable empirical attention and elicited much controversy.[15] Perhaps the most plausible example of a Kuznets curve was in Britain in the nineteenth century, when inequality, as measured by the Gini coefficient, rose from 0.4 in 1823 to 0.627 in 1871, before falling back to 0.443 in 1901.[16] There is also support for the existence of a Kuznets curve in the USA and some other European countries such as France, Germany, and Sweden, while certain other

[13] Braun and Kvasnicka (2013) present an interpretation of female enfranchisement in these countries and states along these lines.

[14] Hicks (2013) presents evidence that spillover effects from other countries and GDP per capita are relevant in explaining female enfranchisement.

[15] Kanbur (2012) provides a recent evaluation of the hypothesis.

[16] See, for example, table 1 in Acemoglu and Robinson (2002, p. 187).

countries in Europe (Norway and the Netherlands) and Asia (Japan, South Korea, and Taiwan) display monotonically declining inequality. However, Van Zanden (1995) suggests that some of the evidence for monotonically declining inequality in Europe arises simply because the observations employed are all on the downswing of the Kuznets curve, and argues for a 'super Kuznets curve' with an upwards phase in inequality from the sixteenth to nineteenth centuries. He provides evidence to this effect from the Netherlands and Northern Italy.

A strategic delegation based explanation for the relationships between the time paths of output, the franchise and inequality is presented by Acemoglu and Robinson (2002). They introduce a non-convexity into the production technology of a dynamic model with a revolutionary threat, and their analysis admits several possible time paths for inequality, including the possibility of a Kuznets curve. This arises when the rich are initially able to overcome the non-convexity in the production technology and accumulate capital while the poor cannot, leading to a phase of increasing inequality. Eventually the increasing prosperity of the rich enables the poor to threaten revolution credibly, and the rich respond to this revolutionary threat by enfranchising the poor. However, once enfranchised the poor instigate a redistributional regime that allows them to overcome the non-convexity in production, and they too begin to accumulate, hence the economy passes into a phase of decreasing inequality.

We show that our information based model of dynamic enfranchisement is also able to explain several of the observed time paths of inequality, including Kuznets curve type behaviour. In the model of dynamic enfranchisement presented there are two influences on inequality; output growth which accrues primarily to the rich enfranchised portion of the population and hence tends to increase inequality, and franchise extensions which increase the proportion of the population sharing in incremental output and hence tend to reduce inequality. These conflicting effects can lead to oligarchic regimes with constant levels of enfranchisement in which inequality can be either declining or increasing, or to a regime of monotonic franchise expansions in which inequality must initially increase but will finally fall as universal suffrage is approached. This explanation seems compatible with the time paths of inequality and franchise extension in nineteenth-century Britain, although of course it focuses on just one out of what must be a myriad of other sources of inequality, by assuming just one kind of inequality is possible.

VI. DISCUSSION

The underlying mechanism behind our results is that more voters mean that better decisions are made. As far as we are aware, the Condorcet Jury Theorem has not so far been suggested as an explanation for extensions of the franchise. For this to be a valid explanation, it is necessary that expanding the size of the electorate benefits those already enfranchised in some way, typically by raising the total output of the economy. This in turn may come about if franchise extension improves (on average) the decisions that are made by the government. In the simple model presented above, the information aggregation mechanism follows directly from the Jury Theorem if there is direct democracy; however there are a number of alternative ways in which this can come about:

(1) The central insight of the theorem is that more voters imply that better decisions are made; in an indirect democracy, where decision makers are elected, it would be interpreted as implying that if there are more voters then the decision makers that are elected tend to make better choices. This could be through a selection mechanism – if there are more voters then, on average, better decision makers are chosen (or whose platforms contain more 'correct' decisions), or through an incentive mechanism – more voters mean that existing decision makers have an incentive to make better decisions.

(2) Another important mechanism arises if an expansion in the number of voters also means an expansion in the number of people who can be elected.[17] An expansion in the number of potential decision makers may allow talented individuals who were previously unable to become part of the government to do so. Also, it can be argued that greater diversity in a group improves group decision making. So if an expansion of the franchise means that a greater diversity of individuals are included in the pool of potential decision makers, this may enhance decision making.
(3) Decision makers need appropriate information, and one source of information for elected decision makers is the electorate. If there are a larger number of voters per elected decision maker, this may enhance both the amount and quality of information they receive. There are a large number of issues on which constituents may contact their elected representatives; on some issues few constituents might contact them. Condorcet-like effects may occur if an expansions of the number of voters increase the number who communicate with them on particular issues.
(4) If there are more voters, they may demand more information. So more information may be supplied; there may be a fixed cost of conveying information, so setting up a means of communication to enable voters to receive relevant information may encourage the transmission of other information which may have beneficial spillover effects on to economic growth. (More concretely, the idea is that enfranchisement might spur the growth of newspapers and other means of communication.)

The idea that more voters raise output may be related to the idea that a larger population promotes technical progress. The latter idea (e.g. Kremer, 1993) is a feature of many endogenous growth models and can be justified by supposing 'that each person's chance of inventing something is independent of population, so that total research output increases in proportion to population' (p. 712). Our idea in this paper is similar, but supposes that for the reasons discussed above, the chances of an invention or productive decision actually being implemented depends (at least to some extent) on the size of the electorate as well as that of the population.

There is also some literature (e.g., Page, 2006) which argues that diversity is a desirable feature of a decision making body.[18] Expanding the franchise *may* be a way of increasing the diversity of either the electorate or of decision makers in government, and hence produce benefits in this way. In summary, then, we believe there are a number of ways in which a larger electorate might raise output; some, but not all, may presuppose a heterogeneous electorate.

As an example of how an information aggregation approach might enhance our understanding of important episodes of franchise extension, consider the Great Reform Act of 1832 in Great Britain. The prevailing opinion is that the threat of revolution was the main motivation for the reform (e.g., Acemoglu and Robinson, 2006, p. 3) and this is not an opinion from which

[17] It is usually, but not always, the case that when a group is enfranchised, it also gains the right to stand for election. One exception is New Zealand, where women were granted the right to vote in 1893, but only gained the right to stand for election in 1919. A curious anomaly in the other direction is provided by the United Kingdom between 1918 and 1928, where women between the ages of 21 and 30 could not vote but could stand for parliament.

[18] For example, it has been argued that lack of diversity in the decision making group may have been responsible for the Bay of Pigs fiasco in 1961 (e.g. Surowiecki, 2004, pp. 36–37). As an example of how diversity can contribute positively, consider the popular UK TV programme 'University Challenge', a general-knowledge contest between universities with each university represented by a four-member team. How should a university choose its team? One possibility might be to hold a general-knowledge contest amongst its students, and pick the four students who show the greatest general knowledge. But such a process might pick students whose strengths and weaknesses overlap. Suppose there are four topics on which questions can be asked: literature, sport, history and science. It is easy to see that a better policy might be to pick the students with the greatest knowledge in each of these subject areas.

we would want to dissent. But the reform would be extremely puzzling if this were the *only* motive for its adoption. The expansion of the franchise was in fact extremely modest and did not seem to enfranchise those who were most likely to rebel, although perhaps it may have deterred revolution to some extent by increasing the number of those with a stake in the current regime.[19] However, the 'information aggregation' approach we take in this paper suggests several more possible reasons for the reform. First, the Reform Act enfranchised a number of cities such as Birmingham and Manchester which were previously completely unrepresented. Information about conditions in these cities was presumably not therefore contained in the information that was gathered by political representatives. Second, the British economy in the early nineteenth century was in the throes of the Industrial Revolution, changing from a largely agricultural to a more industrialised economy. The ruling elite prior to 1832 were mainly landowners, and perhaps not well suited to undertaking the many complicated governmental decisions involved in the transformation of the economy. By expanding the number and type of people eligible for inclusion in the ruling elite, the reform enabled much better decisions to be undertaken. Third, the sizes of individual constituencies were extremely small by today's standards. Even after the Act, there were on average approximately just 1200 voters for each member of parliament as opposed to about 46,000 today. The transmission of information was considerably more difficult and costly than it is now, and one might conjecture that information was also much more differentiated. So an expansion of the number of voters could well have significantly affected the quality and quantity of relevant information transmitted by constituents to their MPs, and therefore might have improved their decision making.

It is of course extremely difficult to evaluate how important the effects we have discussed in this section are. We do not attempt to do so in this paper. Instead, our aim is to argue that this type of explanation of franchise extension is plausible, to derive some implications of a simple formalization of this approach, and to suggest that a fair number of episodes of franchise extension may then be readily explained.

VII. CONCLUSIONS AND POSSIBLE EXTENSIONS

In this paper we have argued that a theory of franchise extensions based on the idea that voting is an information aggregation mechanism in the spirit of Condorcet's Jury Theorem can explain the 'stylized facts' of enfranchisement as well as theories based on strategic delegation. The information aggregation approach explains when political power will remain in the hands of a dictator or small elite and when it will be decentralized to the point of universal suffrage.

We exploit our basic theoretical structure to investigate two further questions. First, we look at the effects of external shocks to the time path of the franchise, and are able to use this to show that such shocks can trigger franchise expansions by shifting the system from a regime of stopping states to a regime of monotonic franchise expansions. This is shown to provide a very plausible explanation for the episodes of female enfranchisement clustered around the ends of the two world wars. Second, we are also able to analyse some features of the dynamic behaviour of inequality. We can show that the Gini coefficient for this economy is increasing along an expansionary time path if the franchise is small and is decreasing as universal suffrage is approached. These are clearly necessary conditions for inequality to follow the classic 'humped' shape of the Kuznets curve.

[19]The percentage of the adult population enfranchised increased from approximately 8 to 10 percent, with all of those enfranchised being property holders.

The information aggregation approach might be further applied to give insight into the question whether there should be further extensions of the franchise. Although many countries now have what they describe as 'universal' suffrage, in reality some groups are always excluded. For example, all countries have a minimum age requirement for voting (usually, but not always, 18), prisoners are often excluded from voting, and countries differ in the extent to which they allow non-citizens to vote.[20] In deciding whether to extend the franchise to any of these groups, the relevant question is whether the benefits to society in terms of improved decision making exceed the costs. However, from society's point of view, the fact that newly enfranchised groups may obtain a greater share of output is not necessarily a cost, but will depend on how society values the gains of those newly enfranchised and the losses of the rest of the population.

The information aggregation approach can explain features of democracy that other approaches find difficult to explain. For example, it is difficult to explain why vote trading should be illegal in democracies using conventional approaches. But it can be shown using the information aggregation approach that vote trading may result in relevant information being lost and lead to worse decision making (Piketty, 1999, pp. 797–98.)

There are several directions in which the information aggregation approach to franchise extension might be further extended. In the analysis above it is assumed that (except for the section on female enfranchisement), all agents are homogeneous. Introducing heterogeneity of information into the structure would allow an investigation of who gets enfranchised first and why. It would also allow for the possibility of decreases in the franchise as some well informed agents may be secure from being disenfranchised. It might also allow for some notion of differential representation, although individuals or groups that are particularly well informed (and hence contribute more to output) may also be able to claim a greater proportional share of output, which may not necessarily be desirable.

One fascinating possible extension concerns the relationship between education and franchise extension. There is evidence (based on post World War II data) that education 'causes' democratization (e.g. Glaeser et al., 2007), but this does not seem to be compatible with (for example) the history of franchise extension in the United Kingdom, in which two major Reform Acts (in 1832 and 1867) occurred before there was any significant public investment in education, which started in about 1870. (See Galor and Moav, 2006, pp. 88–91 for historical evidence on education in the UK.) However, by 1900 school enrollment of 10-year-olds was 100 percent as was male literacy (p. 90), so, just before the next Reform Act was passed, in 1918, a significant fraction of the unefranchised adult population was educated, at least in the sense of being literate and possessing basic skills. An information aggregation approach suggests that there may be greater benefits in extending the franchise to the educated unfranchised than to the uneducated unfranchised and this may be a factor explaining the franchise extensions of 1918 and 1928. Another factor may be that the educated may feel more 'alienated' than the uneducated if they do not have the vote. So exploring the relationship between enfranchisement and education in an extension of the current model would seem to be an obvious idea for further research. We do not believe this approach, though, is so relevant in explaining the earlier franchise extensions of the nineteenth century; democratization is undoubtedly a complex, multi-faceted process and factors which are relevant for some franchise extensions may not be relevant for others.

[20]For example, New Zealand extended the franchise to non-citizen residents in 1975. On the other hand, voting in the United States has, by and large, been confined to citizens. But this has not always been the case and there has been some discussion of whether the franchise should be extended to non-citizens. The United Kingdom falls somewhat between these two extremes, allowing resident Commonwealth and Irish citizens to vote in all elections and resident European Union citizens to vote in all except parliamentary elections.

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APPENDIX

Proof of Proposition 1

To derive the necessary conditions for an interior optimum in the steady state, we first of all differentiate the right-hand side of (3) with respect to $v(t_0)$ and equate to zero to obtain:

$$\left[\frac{\frac{\partial f(v(t_0))}{\partial v(t_0)}v(t_0) - f(v(t_0))}{v(t_0)^2}\right]c(t_0) + \delta\lambda\left[\frac{f(\overline{v})}{\overline{v}}\right]\left[c(t_0)\frac{\partial f(v(t_0))}{\partial v(t_0)}\right]$$
$$+\ \delta^2\lambda^2\left[\frac{f(\overline{v})}{\overline{v}}\right][f(\overline{v})]\left[c(t_0)\frac{\partial f(v(t_0))}{\partial v(t_0)}\right] + \cdots = 0.$$

Evaluating this expression at $v(t_0) = \overline{v}$ generates

$$\left[\frac{\frac{\partial f(\overline{v})}{\partial \overline{v}}\overline{v} - f(\overline{v})}{\overline{v}^2}\right]c(t_0) + \delta\lambda\left[\frac{f(\overline{v})}{\overline{v}}\right]\left[c(t_0)\frac{\partial f(\overline{v})}{\partial \overline{v}}\right] + \delta^2\lambda^2\left[\frac{f(\overline{v})}{\overline{v}}\right][f(\overline{v})]\left[c(t_0)\frac{\partial f(\overline{v})}{\partial \overline{v}}\right] + \cdots = 0$$

which can be simplified to yield

$$\left[\frac{\frac{\overline{v}}{f(\overline{v})} - \frac{1}{\frac{\partial f(\overline{v})}{\partial \overline{v}}}}{\overline{v}}\right] + \delta\lambda + \delta^2\lambda^2 \left[f(\overline{v})\right] + \cdots = 0.$$

So

$$\frac{\overline{v}}{f(\overline{v})}\frac{\partial f(\overline{v})}{\partial \overline{v}} + \frac{\delta\lambda\overline{v}\frac{\partial f(\overline{v})}{\partial \overline{v}}}{1 - \delta\lambda f(\overline{v})} = 1.$$

Using the expression for the efficiency elasticity of the information aggregation process given in the text, we obtain

$$\varepsilon(\overline{v})\left[1 + \frac{\delta\lambda f(\overline{v})}{1 - \delta\lambda f(\overline{v})}\right] = \varepsilon(\overline{v})\left[\frac{1}{1 - \delta\lambda f(\overline{v})}\right] = 1.$$

This gives us equation (4). A steady-state corner solution must then involve

$$\varepsilon(0)\left[\frac{1}{1 - \delta\lambda f(0)}\right] \leq 1$$

or

$$\varepsilon(n)\left[\frac{1}{1 - \delta\lambda f(n)}\right] \geq 1.$$

Hence the conclusions in the text follow.

Proof of Proposition 2

Suppose we write the first-order condition as

$$\varepsilon(\overline{v}) + \delta\lambda f(\overline{v}) - 1 \equiv F(\overline{v}, \delta, \lambda, \alpha) = 0.$$

Then the second-order condition is

$$F_{\overline{v}}(\overline{v}, \delta, \lambda, \alpha) < 0.$$

Applying the implicit function theorem we have:

$$\frac{d\overline{v}}{d\delta} = -\frac{F_\delta}{F_{\overline{v}}} \text{ and } \frac{d\overline{v}}{d\lambda} = -\frac{F_\lambda}{F_{\overline{v}}}$$

Now $F_{\overline{v}} < 0$ from the second-order condition and it is clear that $F_\delta > 0$ and $F_\lambda > 0$, so it follows immediately that $\frac{d\overline{v}}{d\delta} > 0$ and $\frac{d\overline{v}}{d\lambda} > 0$ as claimed by the proposition.

Proof of Proposition 3

Again using the implicit function theorem, we obtain

$$\frac{d\overline{v}}{d\alpha} = -\frac{F_\alpha}{F_{\overline{v}}} = -\frac{\partial\varepsilon(\overline{v})/\partial\alpha + \delta\lambda f_\alpha}{F_{\overline{v}}}$$

The denominator of this expression is negative from the second-order condition, so the sign of the expression is the same as that of F_α. This can be written:

$$F_\alpha = \overline{v}\left[\frac{f f_{\overline{v}\alpha} - f_{\overline{v}} f_\alpha}{f^2}\right] + \delta\lambda f_\alpha = \frac{\overline{v} f_{\overline{v}\alpha}}{f} - \frac{f_\alpha}{f}\left[\varepsilon - \delta\lambda f\right].$$

Using the first-order condition, we can write:

$$F_\alpha = \frac{\overline{v} f_{\overline{v}\alpha}}{f} - \frac{f_\alpha}{f}\left[2\varepsilon - 1\right].$$

The proposition follows immediately: (i) if $f_\alpha > 0$ and $f_{\overline{v}\alpha} = 0$, then the sign of F_α is clearly the same as that of $1 - 2\varepsilon$; (ii) if $f_\alpha = 0$ and $f_{\overline{v}\alpha} > 0$, then F_α is clearly positive.

Proof of Proposition 4

If $v(t+1) < v(t)$, we can write

$$V(v(t), c(t)) = \frac{f(v(t))c(t)}{v(t)} + \delta\frac{v(t+1)}{v(t)}V(v(t+1, c(t+1)) = \frac{f(v(t))c(t)}{v(t)} + \delta\frac{v(t+1)}{v(t)}\left\{\frac{f(v(t+1))c(t+1)}{v(t+1)} + \delta\min\left[\frac{v(t+2)}{v(t+1)}, 1\right]V(v(t+2,)c(t+2))\right\}$$

It is clear that the expression is increasing in $v(t+1)$, implying that the franchise cannot be decreasing anywhere.

Derivation of the dynamics of enfranchisement: expression (8).

$$\begin{aligned}\Pi(t) &\equiv \left[\frac{f(v(t))}{v(t)}\right]c(t) + \delta\lambda\left[\frac{f(v(t+1))}{v(t+1)}\right][c(t)f(v(t))] \\ &\quad + \delta^2\lambda^2\left[\frac{f(v(t+2))}{v(t+2)}\right][f(v(t+1))][c(t)f(v(t))] + \cdots \\ &= c(0)\sum_{t=0}^{\infty}\left[\frac{\delta^t\lambda^t}{v(t)}\right]\left[\prod_{j=0}^{t} f(v(j))\right]\end{aligned}$$

So

$$\begin{aligned}\Pi(t) &\equiv \left[\frac{f(v(t))}{v(t)}\right]c(t) \\ &\quad + \frac{\delta\lambda f(v(t))c(t)f(v(t+1))}{v(t+1)} + \frac{\delta^2\lambda^2 f(v(t))c(t)f(v(t+1))f(v(t+2))}{v(t+2)} + \cdots\end{aligned}$$

$$\begin{aligned}\Pi(t+1) &\equiv \left[\frac{f(v(t+1))}{v(t+1)}\right]c(t+1) + \frac{\delta\lambda f(v(t+1))c(t+1)f(v(t+2))}{v(t+2)} \\ &\quad + \frac{\delta^2\lambda^2 f(v(t+1))c(t+1)f(v(t+2))f(v(t+3))}{v(t+3)} + \cdots\end{aligned}$$

hence

$$\begin{aligned}&\left[\frac{\left(\frac{\partial f(v(t))}{\partial v(t)}\right)v(t) - f(v(t))}{v(t)^2}\right] + \delta\lambda\left[\frac{f(v(t+1))}{v(t+1)}\right]\left(\frac{\partial f(v(t))}{\partial v(t)}\right) \\ &\quad + \delta^2\lambda^2\left[\frac{f(v(t+1))f(v(t+2))}{v(t+2)}\right]\left(\frac{\partial f(v(t))}{\partial v(t)}\right) + \cdots = 0\end{aligned}$$

$$\left[\frac{\left(\frac{\partial f(v(t+1))}{\partial v(t+1)}\right)v(t+1)-f(v(t+1))}{v(t+1)^2}\right]+\delta\lambda\left[\frac{f(v(t+2))}{v(t+2)}\right]\left(\frac{\partial f(v(t+1))}{\partial v(t+1)}\right)$$
$$+\delta^2\lambda^2\left[\frac{f(v(t+2))f(v(t+3))}{v(t+3)}\right]\left(\frac{\partial f(v(t+1))}{\partial v(t+1)}\right)+\cdots=0$$

so

$$\left[\frac{\left(\frac{\partial f(v(t))}{\partial v(t)}\right)v(t)-f(v(t))}{v(t)^2\left(\frac{\partial f(v(t))}{\partial v(t)}\right)}\right]+\delta\lambda\left[\frac{f(v(t+1))}{v(t+1)}\right]$$
$$+\delta^2\lambda^2\left[\frac{f(v(t+1))f(v(t+2))}{v(t+2)}\right]+\cdots=0$$
$$\left[\frac{\left(\frac{\partial f(v(t+1))}{\partial v(t+1)}\right)v(t+1)-f(v(t+1))}{v(t+1)^2\left(\frac{\partial f(v(t+1))}{\partial v(t+1)}\right)}\right]+\delta\lambda\left[\frac{f(v(t+2))}{v(t+2)}\right]$$
$$+\delta^2\lambda^2\left[\frac{f(v(t+2))f(v(t+3))}{v(t+3)}\right]+\cdots=0.$$

Multiplying the second expression by $\delta\lambda f(v(t+1))$ gives

$$\delta\lambda f(v(t+1))\left[\frac{\left(\frac{\partial f(v(t+1))}{\partial v(t+1)}\right)v(t+1)-f(v(t+1))}{v(t+1)^2\left(\frac{\partial f(v(t+1))}{\partial v(t+1)}\right)}\right]+\delta^2\lambda^2\left[\frac{f(v(t+1))f(v(t+2))}{v(t+2)}\right]$$
$$+\delta^3\lambda^3\left[\frac{f(v(t+1))f(v(t+2))f(v(t+3))}{v(t+3)}\right]+\cdots=0$$

implying

$$\left[\frac{\left(\frac{\partial f(v(t))}{\partial v(t)}\right)v(t)-f(v(t))}{v(t)^2\left(\frac{\partial f(v(t))}{\partial v(t)}\right)}\right]+\delta\lambda\left[\frac{f(v(t+1))}{v(t+1)}\right]$$
$$=\delta\lambda f(v(t+1))\left[\frac{\left(\frac{\partial f(v(t+1))}{\partial v(t+1)}\right)v(t+1)-f(v(t+1))}{v(t+1)^2\left(\frac{\partial f(v(t+1))}{\partial v(t+1)}\right)}\right]\left[\frac{\left(\frac{\partial f(v(t))}{\partial v(t)}\right)v(t)-f(v(t))}{v(t)^2\left(\frac{\partial f(v(t))}{\partial v(t)}\right)}\right]$$
$$=-\delta\lambda f(v(t+1))\left[\frac{f(v(t+1))}{v(t+1)^2\left(\frac{\partial f(v(t+1))}{\partial v(t+1)}\right)}\right]$$

so as in the text we get

$$\frac{v(t+1)}{v(t)}=\delta\lambda f(v(t+1))\left[\frac{1}{1-\varepsilon^v(t)}\right]\left[\frac{\varepsilon^v(t)}{\varepsilon^v(t+1)}\right]$$

Proof of Proposition 5

Part (i) asserts monotonic convergence to universal suffrage. Rewriting the first-order condition (6) we have

$$\left[\frac{\varepsilon^{v}(t)-1}{\frac{v(t)^2}{f(v(t))}}\right]+\delta\lambda\left[\frac{f(v(t+1))}{v(t+1)}\right]\left(\frac{\partial f(v(t))}{\partial v(t)}\right)$$
$$+\delta^2\lambda^2\left[\frac{f(v(t+1))f(v(t+2))}{v(t+2)}\right]\left(\frac{\partial f(v(t))}{\partial v(t)}\right)+\cdots=0$$

or

$$\left[\frac{\varepsilon^{v}(t)-1}{v(t)}\right]+\delta\lambda\left[\frac{f(v(t+1))}{v(t+1)}\right]\varepsilon^{v}(t)$$
$$+\delta^2\lambda^2\left[\frac{f(v(t+1))f(v(t+2))}{v(t+2)}\right]\varepsilon^{v}(t)+\cdots=0$$

a necessary condition for which to hold is $\varepsilon^{v}(t)\in[0,1)$. The second-order condition for an interior maximum requires

$$\frac{\partial^2 V(v(t),c(t))}{\partial v(t)^2}=\left(\frac{\partial\varepsilon^{v}(t)}{\partial v(t)}v(t)-\varepsilon^{v}(t)+1\right)\left(\frac{1}{v(t)^2}\right)$$
$$+\frac{\partial\varepsilon^{v}(t)}{\partial v(t)}\left[\delta\lambda\left[\frac{f(v(t+1))}{v(t+1)}\right]+\delta^2\lambda^2\left[\frac{f(v(t+1))f(v(t+2))}{v(t+2)}\right]+\cdots\right]<0$$

a necessary condition for which is $\frac{\partial\varepsilon^{v}(t)}{\partial v(t)}<0$. Applying these two pieces of information to the difference equation (8) we may deduce $\frac{\varepsilon^{v}(t)}{\varepsilon^{v}(t+1)}>1$ and $\frac{1}{1-\varepsilon^{v}(t)}>1$ so we may immediately see that $\delta\lambda f(v(0))>1$ is an appropriate sufficient (but not necessary) condition. Part (ii) states that the interval $v(t)\in[0,v^*]$ are stopping states while $v(t)\in(v^*,n]$ is a region of monotonic convergence to universal suffrage. It serves to demonstrate that at an interior steady state $\frac{dv(t+1)}{dv(t)}>1$, so rewriting (8) we have

$$\frac{v(t+1)}{f(v(t+1))}\varepsilon^{v}(t+1)=\delta\lambda\left[\frac{\varepsilon^{v}(t)}{1-\varepsilon^{v}(t)}\right]v(t).$$

Differentiating with respect to $v(t)$ and $v(t+1)$ gives

$$\left[\frac{\left[\varepsilon^{v}(t+1)+\frac{\partial\varepsilon^{v}(t)}{\partial v(t)}v(t+1)\right]f(v(t+1))-\frac{\partial f(v(t))}{\partial v(t)}\varepsilon^{v}(t+1)v(t+1)}{f(v(t+1))^2}\right]dv(t+1)$$
$$=\delta\lambda\left[\frac{\varepsilon^{v}(t)}{1-\varepsilon^{v}(t)}+\left(\frac{\frac{\partial\varepsilon^{v}(t+1)}{\partial v(t+1)}(1-\varepsilon^{v}(t))+\frac{\partial\varepsilon^{v}(t+1)}{\partial v(t+1)}\varepsilon^{v}(t)}{(1-\varepsilon^{v}(t))^2}\right)v(t)\right]dv(t)$$

$$\left[\varepsilon^{v}(t+1)+\frac{\partial\varepsilon^{v}(t)}{\partial v(t)}v(t+1)-\varepsilon^{v}(t+1)^2\right]dv(t+1)$$
$$=\delta\lambda f(v(t+1))\left[\frac{\varepsilon^{v}(t)}{1-\varepsilon^{v}(t)}+\left(\frac{\frac{\partial\varepsilon^{v}(t+1)}{\partial v(t+1)}}{(1-\varepsilon^{v}(t))^2}\right)v(t)\right]dv(t).$$

So

$$\frac{dv(t+1)}{dv(t)} = \frac{\delta\lambda f(v(t+1))\left[\frac{\varepsilon^v(t)}{1-\varepsilon^v(t)} + \left(\frac{\frac{\partial\varepsilon^v(t+1)}{\partial v(t+1)}}{(1-\varepsilon^v(t))^2}\right)v(t)\right]}{\varepsilon^v(t+1) + \frac{\partial\varepsilon^v(t)}{\partial v(t)}v(t+1) - \varepsilon^v(t+1)^2}$$

Now evaluating expression (8) at $v(t) = v(t+1)$ gives

$$1 - \varepsilon^v = \delta\lambda f(v)$$

So evaluating $\frac{dv(t+1)}{dv(t)}$ at $v(t) = v(t+1)$ and using $1 - \varepsilon^v = \delta\lambda f(v)$ gives

$$\begin{aligned}
\left.\frac{dv(t+1)}{dv(t)}\right|_{v(t)=v(t+1)} &= \frac{(1-\varepsilon^v)\left[\frac{\varepsilon^v}{1-\varepsilon^v} + \left(\frac{\frac{\partial\varepsilon^v}{\partial v}}{(1-\varepsilon^v)^2}\right)v\right]}{\varepsilon^v + \frac{\partial\varepsilon^v}{\partial v}v - (\varepsilon^v)^2} \\
&= \frac{\varepsilon^v + \left(\frac{\frac{\partial\varepsilon^v}{\partial v}}{1-\varepsilon^v}\right)v}{\varepsilon^v + \frac{\partial\varepsilon^v}{\partial v}v - (\varepsilon^v)^2} = \frac{\left(\frac{\varepsilon^v + \frac{\partial\varepsilon^v}{\partial v}v - (\varepsilon^v)^2}{1-\varepsilon^v}\right)}{\varepsilon^v + \frac{\partial\varepsilon^v}{\partial v}v - (\varepsilon^v)^2} \\
&= \frac{1}{1-\varepsilon^v} > 1.
\end{aligned}$$

Proof of Proposition 6

Rewriting (8) $\frac{v(t+1)\varepsilon^v(t+1)}{f(v(t+1))} = \delta\lambda\left[\frac{\varepsilon^v(t)}{1-\varepsilon^v(t)}\right]v(t)$ so

$$\left.\frac{dv(t+1)}{d(\delta\lambda)}\right|_{v(t)} = \frac{\left[\frac{\varepsilon^v(t)}{1-\varepsilon^v(t)}\right]v(t)}{\frac{\left(\varepsilon^v(t+1)+v(t+1)\frac{\partial\varepsilon^v(t)}{\partial v(t)}\right)f(v(t+1)) - \frac{\partial f(v(t))}{\partial v(t)}v(t+1)\varepsilon^v(t+1)}{f(v(t+1))^2}} = \frac{\left[\frac{\varepsilon^v(t)}{1-\varepsilon^v(t)}\right]v(t)}{\frac{\varepsilon^v(t+1)+v(t+1)\frac{\partial\varepsilon^v(t)}{\partial v(t)} - (\varepsilon^v(t+1))^2}{f(v(t+1))}}$$

and

$$sgn\left.\frac{dv(t+1)}{d(\delta\lambda)}\right|_{v(t)} = sgn\left[\varepsilon^v(t+1) + v(t+1)\frac{\partial\varepsilon^v(t)}{\partial v(t)} - (\varepsilon^v(t+1))^2\right]$$

We know $\varepsilon^v(t+1)$ is decreasing so if it is convex (sufficient but not necessary condition) then $\varepsilon^v(t+1) + v(t+1)\frac{\partial\varepsilon^v(t)}{\partial v(t)} < 0$ and we have $\left.\frac{dv(t+1)}{d(\delta\lambda)}\right|_{v(t)} < 0$, alternatively if $\varepsilon^v(t+1) + v(t+1)\frac{\partial\varepsilon^v(t)}{\partial v(t)} - (\varepsilon^v(t+1))^2 > 0$ which requires $\varepsilon^v(t+1)$ is decreasing and sufficiently concave then $\left.\frac{dv(t+1)}{d(\delta\lambda)}\right|_{v(t)} > 0$.

Proof of Proposition 7

It suffices to show that the solution to the difference equation (8) when evaluated at an interior steady state $v(t) = v(t+1)$ is decreasing in the vote efficiency parameter (see figure 3). Rewriting (8) appropriately and evaluating at $v(t) = v(t+1) \equiv \overline{v}$ we have

$$1 - \varepsilon^{\alpha\overline{v}} = \delta\lambda f(\alpha\overline{v})$$

where $\varepsilon^{\alpha\overline{v}} \equiv \frac{\partial f(\alpha\overline{v})}{\partial(\alpha\overline{v})}\frac{\alpha\overline{v}}{f(\alpha\overline{v})}$. Now differentiating w.r.t. α and $\overline{v}$ gives

$$-\frac{\partial\varepsilon^{\alpha\overline{v}}}{\partial\alpha}d\alpha - \frac{\partial\varepsilon^{\alpha\overline{v}}}{\partial\overline{v}}d\overline{v} = \delta\lambda\overline{v}\frac{\partial f(\alpha\overline{v})}{\partial(\alpha\overline{v})}d\alpha + \delta\lambda\alpha\frac{\partial f(\alpha\overline{v})}{\partial(\alpha\overline{v})}d\overline{v}$$

so

$$d\overline{v}\left(\delta\lambda\alpha\frac{\partial f(\alpha\overline{v})}{\partial(\alpha\overline{v})}+\frac{\partial\varepsilon^{\alpha\overline{v}}}{\partial\overline{v}}\right)=-d\alpha\left(\delta\lambda\overline{v}\frac{\partial f(\alpha\overline{v})}{\partial(\alpha\overline{v})}+\frac{\partial\varepsilon^{\alpha\overline{v}}}{\partial\alpha}\right)$$

or

$$\frac{d\overline{v}}{d\alpha}=-\left(\frac{\delta\lambda\overline{v}\frac{\partial f(\alpha\overline{v})}{\partial(\alpha\overline{v})}+\frac{\partial\varepsilon^{\alpha\overline{v}}}{\partial\alpha}}{\delta\lambda\alpha\frac{\partial f(\alpha\overline{v})}{\partial(\alpha\overline{v})}+\frac{\partial\varepsilon^{\alpha\overline{v}}}{\partial\overline{v}}}\right)$$

Now

$$\frac{\partial\varepsilon^{\alpha\overline{v}}}{\partial\alpha}=\overline{v}\frac{\partial^2 f(\alpha\overline{v})}{\partial(\alpha\overline{v})^2}\left(\frac{\alpha\overline{v}}{f(\alpha\overline{v})}\right)+\frac{\partial f(\alpha\overline{v})}{\partial(\alpha\overline{v})}\left[\frac{\overline{v}f(\alpha\overline{v})-\alpha\overline{v}^2\frac{\partial f(\alpha\overline{v})}{\partial(\alpha\overline{v})}}{f(\alpha\overline{v})^2}\right]$$

$$\frac{\partial\varepsilon^{\alpha\overline{v}}}{\partial\overline{v}}=\alpha\frac{\partial^2 f(\alpha\overline{v})}{\partial(\alpha\overline{v})^2}\left(\frac{\alpha\overline{v}}{f(\alpha\overline{v})}\right)+\frac{\partial f(\alpha\overline{v})}{\partial(\alpha\overline{v})}\left[\frac{\alpha f(\alpha\overline{v})-\alpha^2\overline{v}\frac{\partial f(\alpha\overline{v})}{\partial(\alpha\overline{v})}}{f(\alpha\overline{v})^2}\right]$$

so

$$\begin{aligned}\frac{d\overline{v}}{d\alpha}&=-\left(\frac{\delta\lambda\overline{v}\frac{\partial f(\alpha\overline{v})}{\partial(\alpha\overline{v})}+\overline{v}\frac{\partial^2 f(\alpha\overline{v})}{\partial(\alpha\overline{v})^2}\left(\frac{\alpha\overline{v}}{f(\alpha\overline{v})}\right)+\frac{\partial f(\alpha\overline{v})}{\partial(\alpha\overline{v})}\left[\frac{\overline{v}f(\alpha\overline{v})-\alpha\overline{v}^2\frac{\partial f(\alpha\overline{v})}{\partial(\alpha\overline{v})}}{f(\alpha\overline{v})^2}\right]}{\delta\lambda\alpha\frac{\partial f(\alpha\overline{v})}{\partial(\alpha\overline{v})}+\alpha\frac{\partial^2 f(\alpha\overline{v})}{\partial(\alpha\overline{v})^2}\left(\frac{\alpha\overline{v}}{f(\alpha\overline{v})}\right)+\frac{\partial f(\alpha\overline{v})}{\partial(\alpha\overline{v})}\left[\frac{\alpha f(\alpha\overline{v})-\alpha^2\overline{v}\frac{\partial f(\alpha\overline{v})}{\partial(\alpha\overline{v})}}{f(\alpha\overline{v})^2}\right]}\right)\\&=-\frac{\overline{v}}{\alpha}<0.\end{aligned}$$

Proof of Proposition 8

Part (i) follows simply from noting that (10) is increasing in $s(t)=f(\overline{v})c(t)$ and that $c(t)=\lambda f(\overline{v})s(t-1)$. Part (ii) requires differentiating expression (10) with respect to $v(t)$ and evaluating the result as $v(t)$ approaches its limits. Differentiating (10) gives

$$\begin{aligned}\frac{dG(v(t))}{dv(t)}=&\frac{\partial f(v(t))}{\partial v(t)}\left(\frac{c(t)}{n}\right)\left(\frac{n-v(t)}{f(v(t))c(t)+mn}\right)\\&-\left(\frac{f(v(t))c(t)}{n}\right)\left(\frac{f(v(t))c(t)+mn+\frac{\partial f(v(t))}{\partial v(t)}c(t)(n-v(t))}{(f(v(t))c(t)+mn)^2}\right)\end{aligned}$$

so writing

$$\begin{aligned}J(v(t))&=sgn\left[\frac{dG(v(t))}{dv(t)}\right]\\&=sgn\left[\frac{\partial f(v(t))}{\partial v(t)}(n-v(t))-f(v(t))\left(\frac{f(v(t))c(t)+mn+\frac{\partial f(v(t))}{\partial v(t)}c(t)(n-v(t))}{(f(v(t))c(t)+mn)}\right)\right].\end{aligned}$$

Evaluating this expression as it approaches its limits gives

$$J(v(t))|_{v(t)\to 0}=sgn\left[\frac{\partial f(v(t))}{\partial v(t)}\right]>0$$

$$J(v(t))|_{v(t)\to n}=sgn\left[-f(v(t))\right]<0$$

as required.